Binomial Theorem

Question1

The value of $^{47}C_4 + \sum_{j=1}^{5} {}^{(52-j)}C_3$ is MHT CET 2025 (5 May Shift 2)

Options:

A. $^{52}C_4$

B. $^{52}C_2$

C. $^{48}C_4$

D. $^{48}C_2$

Answer: A

Solution:

The value of $47C_4 + \sum_{j=1}^{5} 52 - jC_3$ is $52C_4$.

Explanation

- The summation expands: $52 - 1C_3 + 52 - 2C_3 + 52 - 3C_3 + 52 - 4C_3 + 52 - 5C_3 = 51C_3 + 50C_3 + 49C_3 + 48C_3 + 47C_3$.
- Add $47C_4$ to get: $47C_4 + 51C_3 + 50C_3 + 49C_3 + 48C_3 + 47C_3$.
- Using combinations identity (Hockey Stick), this sum simplifies directly to $52C_4$.

the answer is Option A: $^{52}C_4$.

Question2

If $^{n}C_0 + \frac{1}{2}{}^{n}C_1 + \frac{1}{3}{}^{n}C_2 + \ldots\ldots\ldots\ldots\ldots \cdot \frac{1}{n}{}^{n}C_{n-1} + \frac{1}{n+1}{}^{n}C_n = \frac{1023}{10}$ then $n =$ MHT CET 2025 (25 Apr Shift 1)

Options:

A. 7

B. 8

C. 9

D. 10

Answer: C

Solution:

Use

$$\sum_{k=0}^{n} \frac{1}{k+1}\binom{n}{k} = \int_0^1 (1+x)^n dx = \frac{(1+1)^{n+1}-1}{n+1} = \frac{2^{n+1}-1}{n+1}.$$

Set equal to $1023/10$:

$$\frac{2^{n+1}-1}{n+1} = \frac{1023}{10} \Rightarrow 10(2^{n+1}-1) = 1023(n+1).$$

Try options → $n = 9$ satisfies since $2^{10} - 1 = 1023$.

$\boxed{n = 9}$.

Question3

If ${}^{15}C_4 + {}^{15}C_5 + {}^{16}C_6 + {}^{17}C_7 + {}^{18}C_8 = {}^{19}C_r$, then the value of r is equal to MHT CET 2025 (23 Apr Shift 2)

Options:

A. 9 or 10

B. 7 or 12

C. 8 or 10

D. 8 or 11

Answer: D

Solution:

Use the Pascal identity: $\binom{n}{k}+\binom{n}{k+1}=\binom{n+1}{k+1}$.

$$\binom{15}{4}+\binom{15}{5}=\binom{16}{5},$$
$$\binom{16}{5}+\binom{16}{6}=\binom{17}{6},$$
$$\binom{17}{6}+\binom{17}{7}=\binom{18}{7},$$
$$\binom{18}{7}+\binom{18}{8}=\binom{19}{8}.$$

Hence the sum equals $\binom{19}{8}=\binom{19}{11} \Rightarrow r=8 \text{ or } 11$.

Question4

The value of $\frac{{}^{10}\mathrm{C}_{\mathrm{r}}}{{}^{11}\mathrm{C}_{\mathrm{r}}}$, when both the numerator and denominator are at their greatest values, is MHT CET 2023 (10 May Shift 2)

Options:

A. $\frac{6}{11}$

B. $\frac{1}{11}$

C. $\frac{4}{11}$

D. $\frac{3}{11}$

Answer: A

Solution:

Greatest value of $^{-10}\mathrm{C}_{\mathrm{r}}$ is at $\mathrm{r}=5$ Greatest value of $^{11}\mathrm{C}_{\mathrm{r}}$ is at $\mathrm{r}=5 \therefore \quad \frac{{}^{10}\mathrm{C}_{\mathrm{r}}}{{}^{11}\mathrm{C}_{\mathrm{r}}}=\frac{{}^{10}\mathrm{C}_5}{{}^{11}\mathrm{C}_5}=\frac{\frac{10!}{5!5!}}{\frac{11!}{5!6!}}=\frac{6}{11}$

Question5

If ${}^{11}\mathrm{C}_4+{}^{11}\mathrm{C}_5+{}^{12}\mathrm{C}_6+{}^{13}\mathrm{C}_7={}^{14}\mathrm{C}_\mathrm{r}$, then value of r is MHT CET 2021 (24 Sep Shift 1)

Options:

A. 11

B. 14

C. 7

D. 3

Answer: C

Solution:

$$ {}^{11}\mathrm{C}_4+{}^{11}\mathrm{C}_5+{}^{12}\mathrm{C}_6+{}^{13}\mathrm{C}_7={}^{14}\mathrm{C}_\mathrm{r} $$

We know that ${}^{n}C_r+{}^{n}C_{r-1}={}^{n+1}C_r$

$$ \begin{aligned} &\therefore {}^{14}\mathrm{C}_\mathrm{r}=\left({}^{11}\mathrm{C}_4+{}^{11}\mathrm{C}_5\right)+{}^{12}\mathrm{C}_6+{}^{13}\mathrm{C}_7\\ &=\left({}^{12}\mathrm{C}_5+{}^{12}\mathrm{C}_6\right)+{}^{13}\mathrm{C}_7\\ &={}^{13}\mathrm{C}_6+{}^{13}\mathrm{C}_7={}^{14}\mathrm{C}_7\\ &\therefore \mathrm{r}=7 \end{aligned} $$

Question6

The difference between the maximum values of ${}^{6}\mathrm{C}_\mathrm{r}$ and ${}^{n}\mathrm{C}_\mathrm{r}16$, then $\mathrm{n}=$ MHT CET 2021 (23 Sep Shift 1)

Options:

A. 3

B. 5

C. 2

D. 4

Answer: D

Solution:

The maximum value of ${}^6\mathrm{C}_\mathrm{r}$ occurs at $\mathrm{r}=\frac{6}{2}=3$

$$\therefore {}^6\mathrm{C}_3=\frac{6!}{3!3!}=\frac{6\times5\times4}{6}=20$$

As per data given $\left|{}^6\mathrm{C}_3-{}^\mathrm{n}\mathrm{C}_3\right|=16$

$$\therefore {}^\mathrm{n}\mathrm{C}_3=20+16 \text{ or } {}^\mathrm{n}\mathrm{C}_3=20-16$$

If ${}^nC_3=36\Rightarrow\frac{n!}{(n-3)!n!}=36\Rightarrow n(n-1)(n-2)=216$ is not possible for

$$\mathrm{n}\in\mathrm{N}$$

$$\text{If } {}^\mathrm{n}\mathrm{C}_3=4\Rightarrow\frac{\mathrm{n}!}{(\mathrm{n}-3)!3!}=4\Rightarrow\mathrm{n}(\mathrm{n}-1)(\mathrm{n}-2)=24$$

$$\Rightarrow\mathrm{n}=4$$

Question7

The co-efficient of x^6 in the series of e^{2x} is MHT CET 2020 (20 Oct Shift 2)

Options:

A. $\frac{2}{45}$

B. $\frac{7}{45}$

C. $\frac{4}{45}$

D. $\frac{1}{45}$

Answer: C

Solution:

$$e^{2x} = 1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} + \frac{(2x)^6}{6!} + x^6 \rightarrow$$

$$\text{coefficient} = \left(\frac{2^6 \cdot x^6}{6!} = \frac{4}{45}\right.$$
